

# **Breaking RSA is Equivalent to Factoring**

**Divesh Aggarwal and Ueli Maurer**

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CRYPTO 2008 Rump Session.

# **Breaking RSA Generically** **is Equivalent to Factoring**

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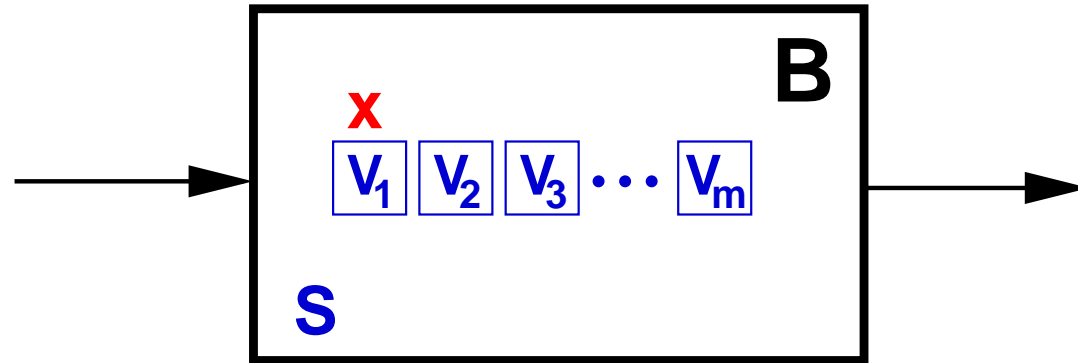
# Generic algorithms

- **Cannot exploit representation of elements, except for trivial properties.**
- **Used to prove lower bounds (e.g. DL, DH, DDH).**
- **Many known algorithms/reductions are generic.**
- **Often modeled as a random mapping [Shoup97].**

# Generic algorithms

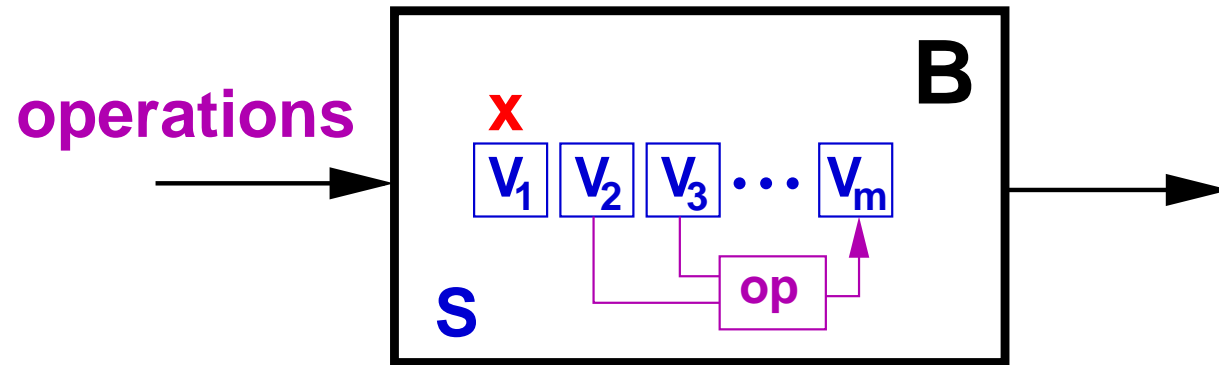
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- **Many known algorithms/reductions are generic.**
- **Often modeled as a random mapping [Shoup97].**
- **Next: simpler and more general abstract model of computation [Mau05].**

# Abstract Model of Computation



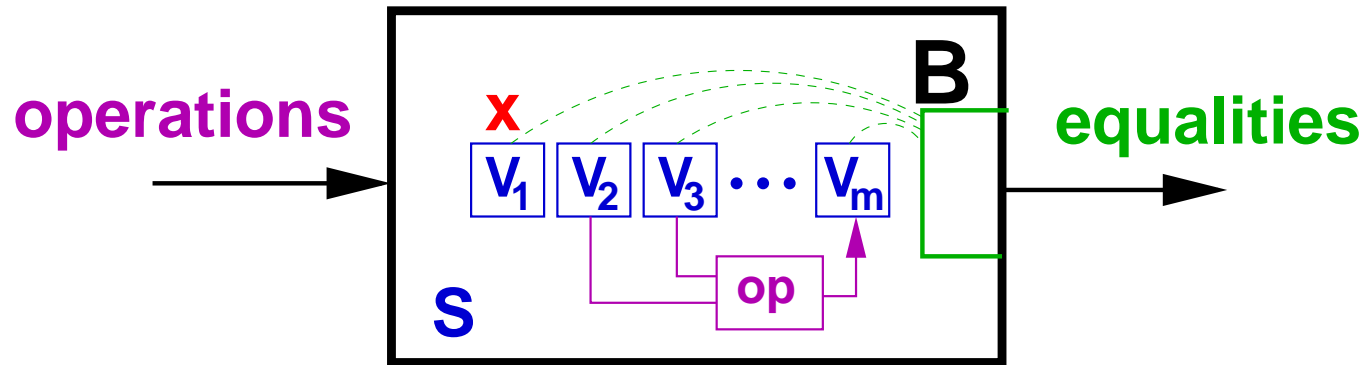
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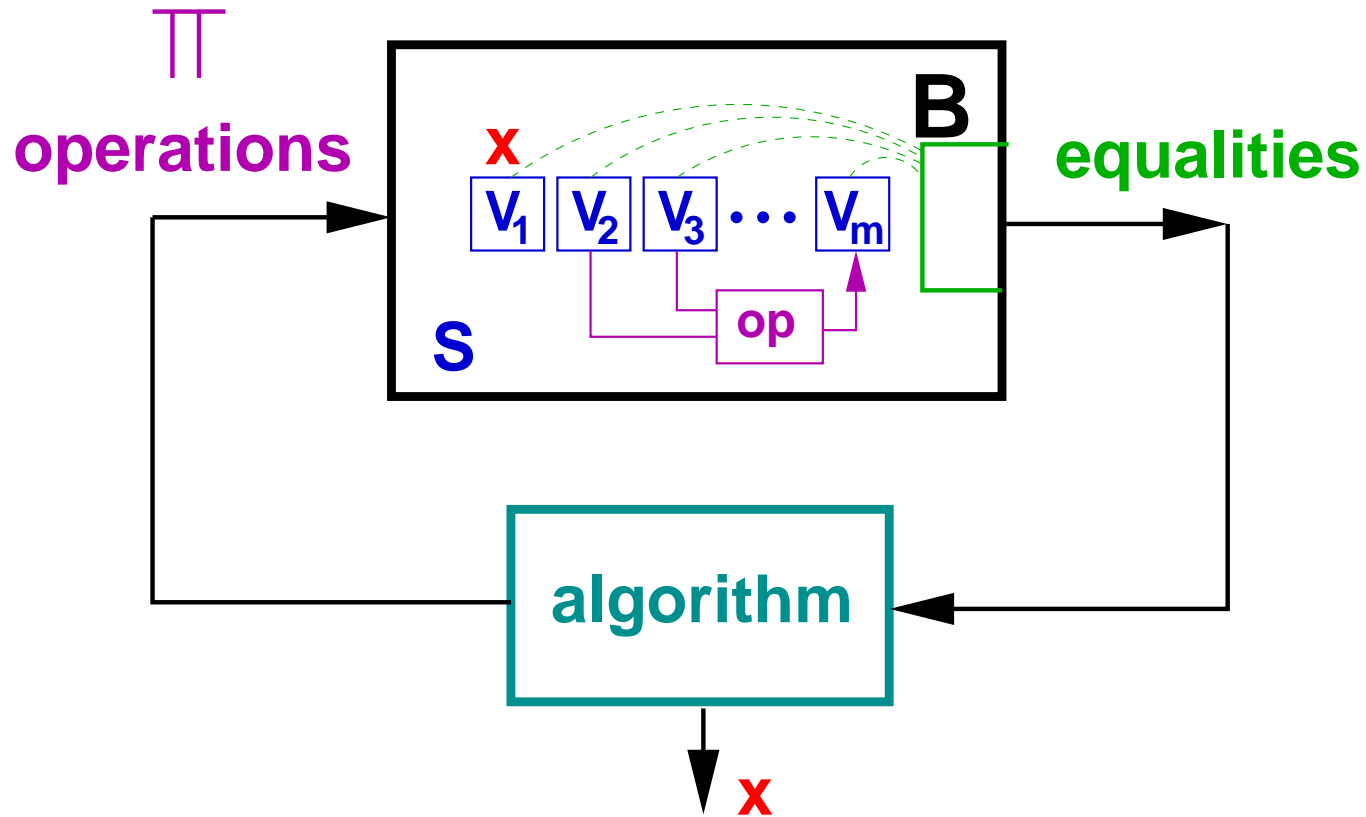
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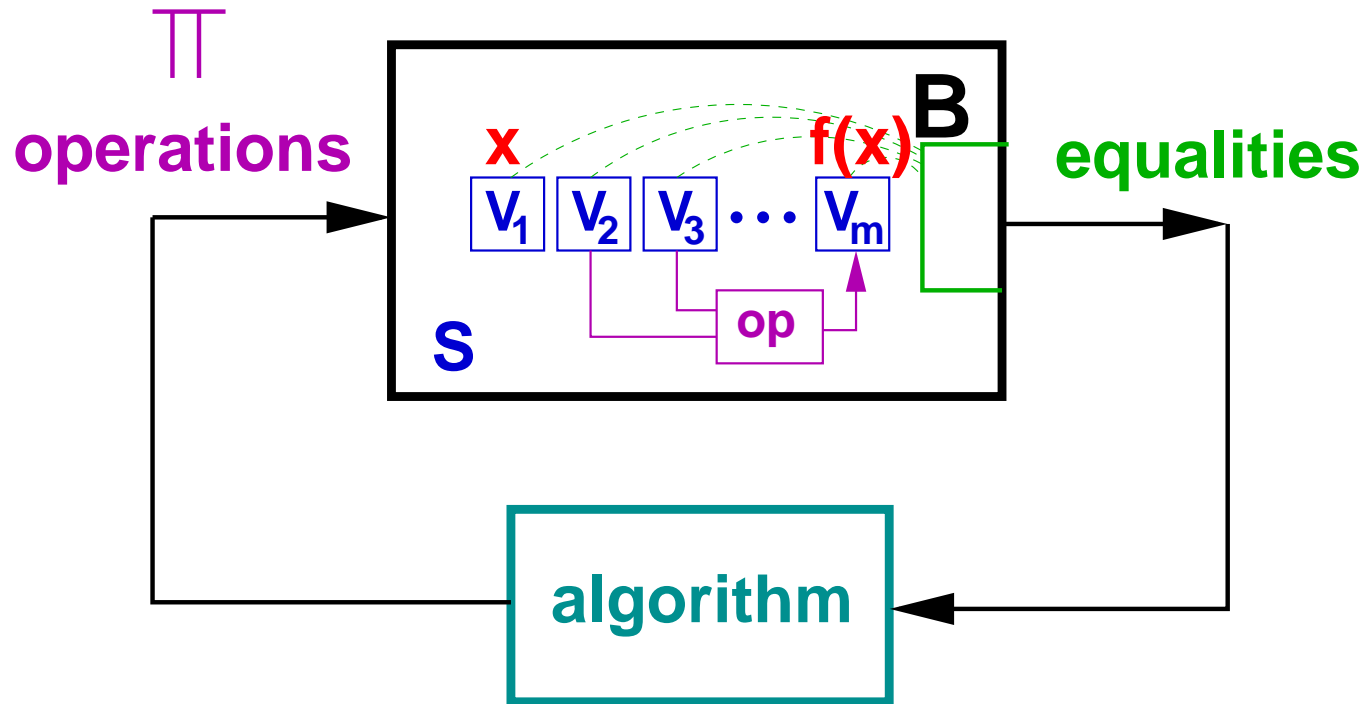
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- Task of **algorithm**: extract  $x$ .

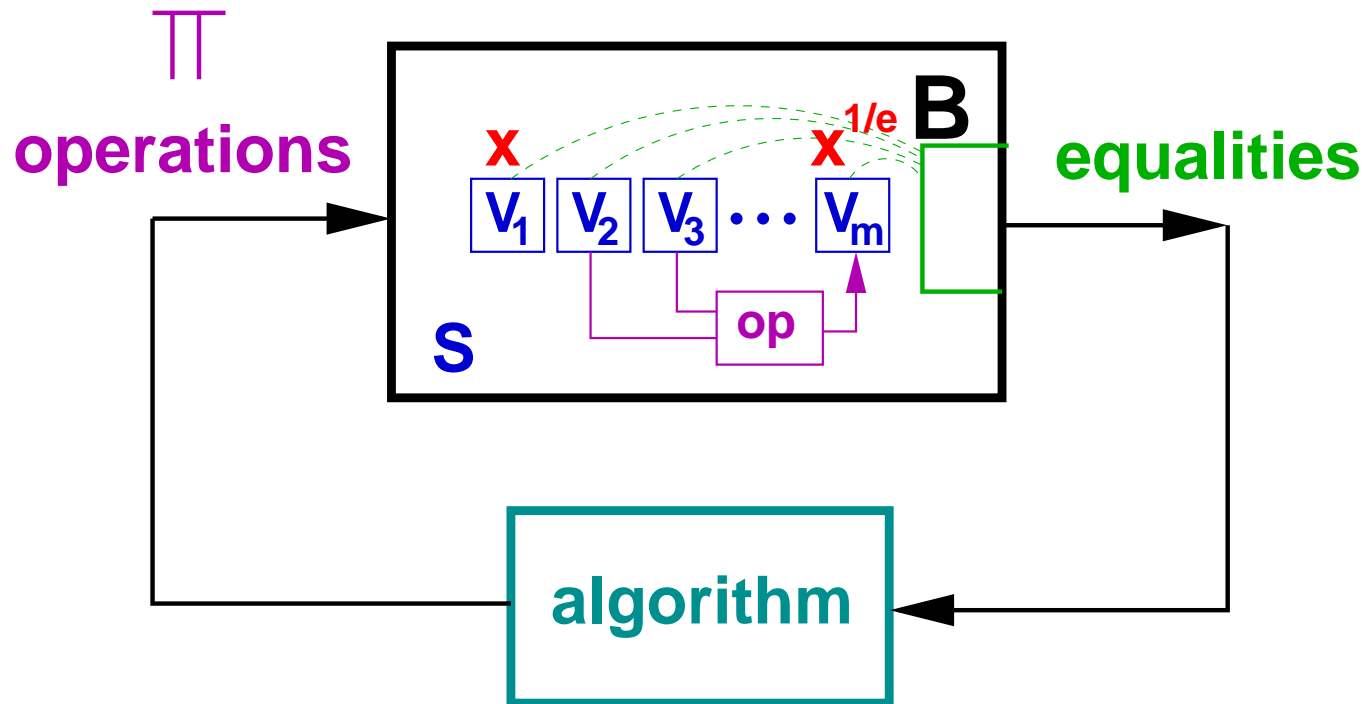


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- **B** allows to perform internal **operations**.
- Possible type of operation: **equality tests**.
- Task of **algorithm**: achieve  $V_m = f(x)$  for some **f**.

# Breaking RSA Generically

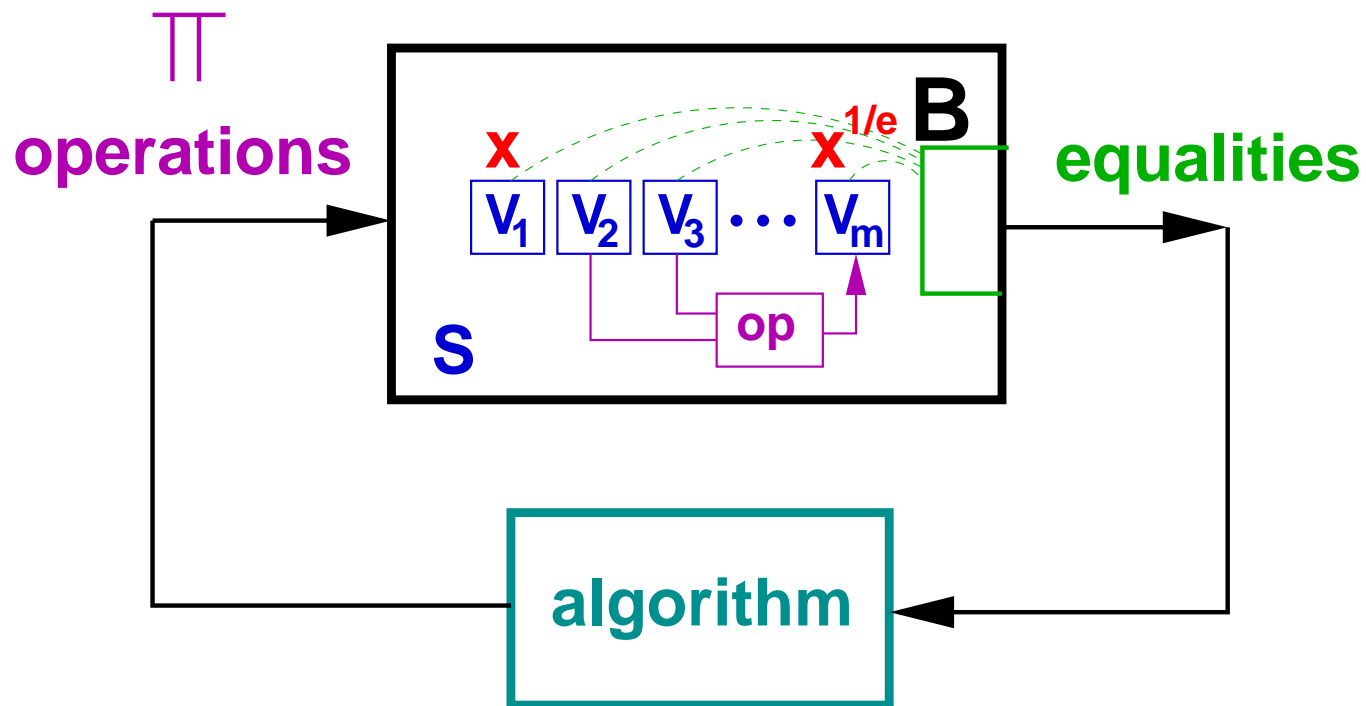


$$\mathbf{S} = \mathbb{Z}_n$$

$$f(x) = \sqrt[e]{x}$$

$$\Pi = \{+, -, *, /, (\cdot)^{-1}, \text{eq?}\}$$

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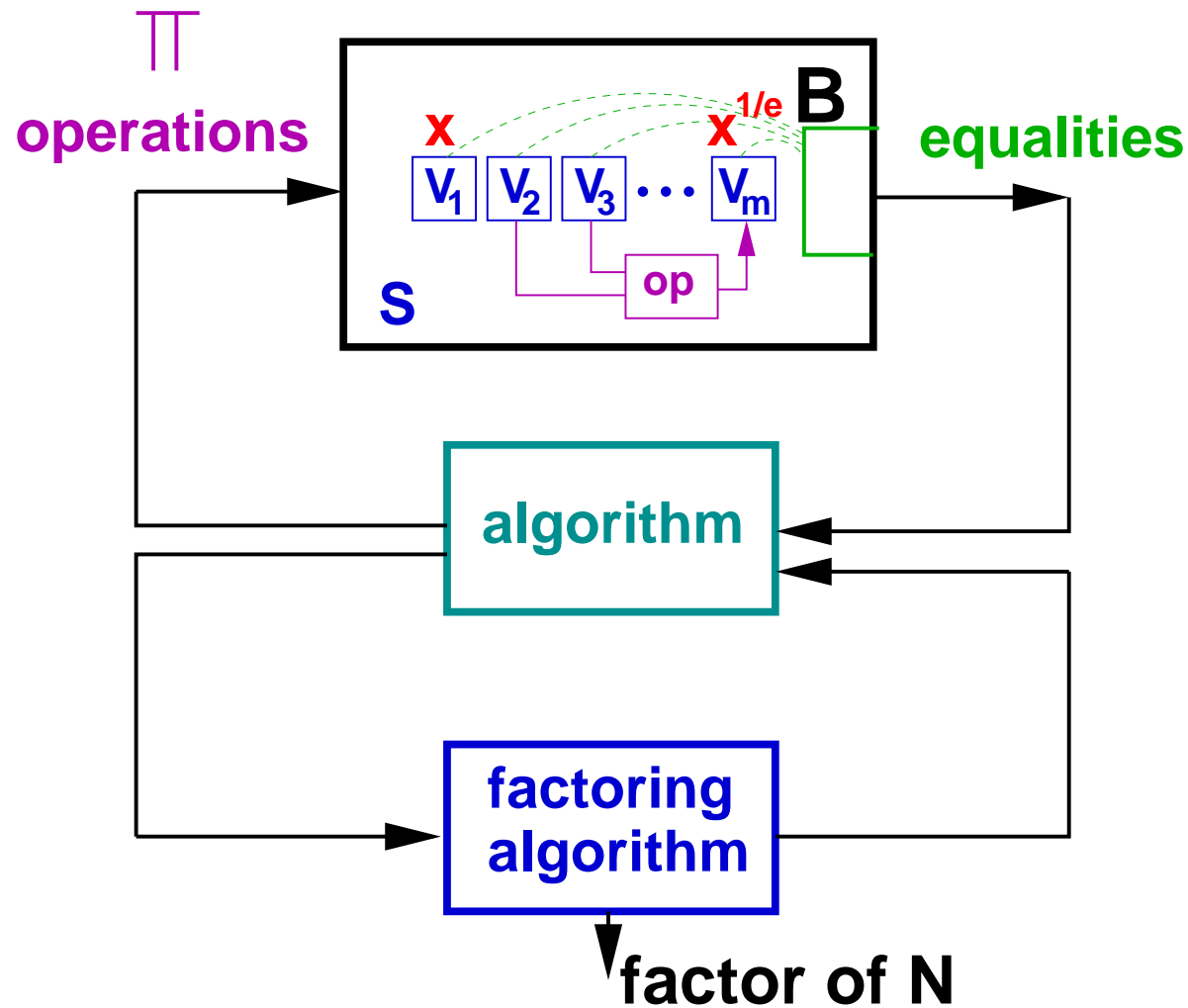
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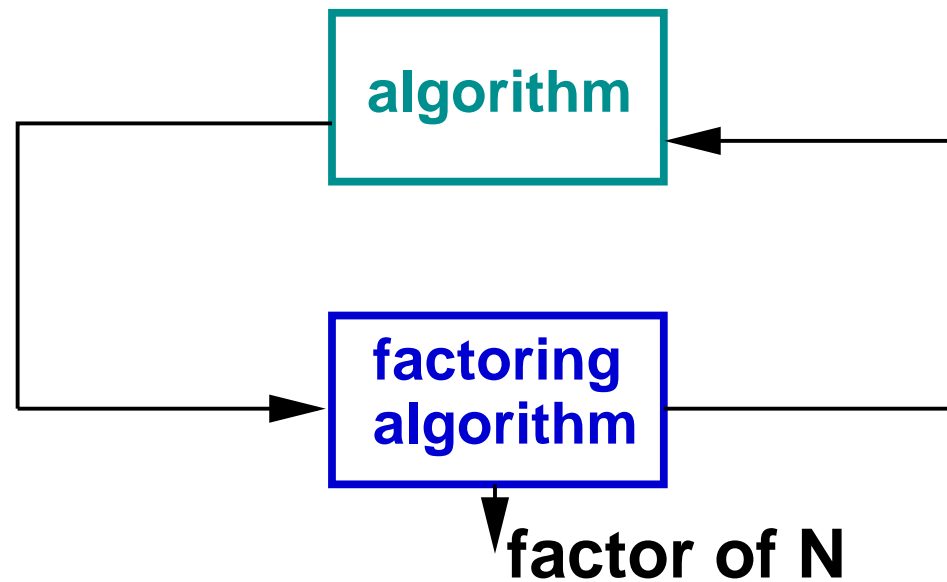
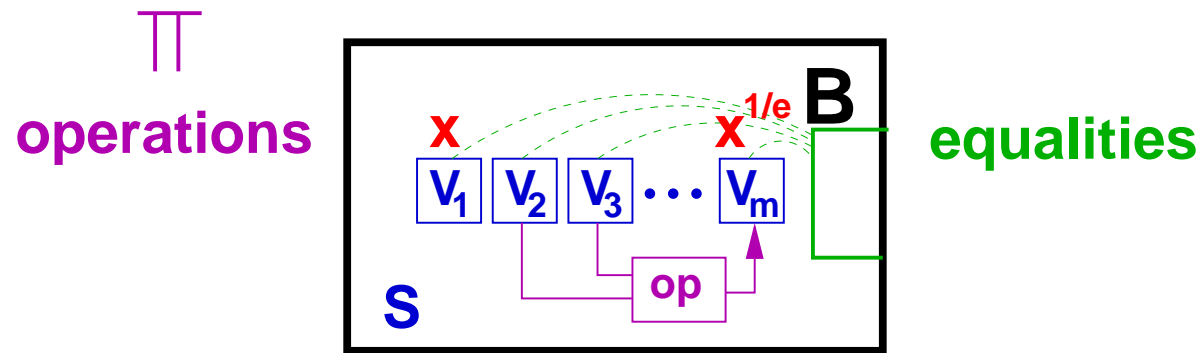
Previous results: [Brown05], [Leander/Rupp06],

**Theorem:** There exists an efficient algorithm which, when given access to any generic  $e$ -th root algorithm, factors  $N$  (with essentially the same success prob.).

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**Paper on e-print, but contains an error!**

**Fixed version (more involved) will soon be on-line.**